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RELAXATIONAL PROPERTIES OF A TURBULENT SHEAR FLOW ACROSS A CYLINDER IN THE PRESENCE OF A PLATE

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With the use of semi-empirical turbulence models to analyze essentially nonequilibrium turbulent boundary layers in the last decade, there have been increasingly frequent complaints about the inadequacy of the traditional approaches that have been employed to solve such problems [1]. One reason for this dissatisfaction is the local nature of the turbulence hypotheses for the external region, which is characterized by the presence of large, long-lived eddies – the main sources of information on perturbations. In connection with this, the Boussinesq approximation proves to be inadequate for the analysis of flows in this region. As regards the internal region – characterized by small-scale turbulence and offering less information on perturbations – the use of the Boussinesq hypothesis is obviously valid. This shows the need to resort to the use of a relaxation theory (heredity theory) based on a new formula for turbulent shear stress that will make it possible to account for the history (memory) of the boundary layer in regard to a given disturbance. By relaxation, in the process by which some physical quantity derived from the equilibrium state returns to this state [2].

The first attempt to account for relaxation processes in turbulent shear flows was made by Hinze [3] by means of an equation that was also derived and analyzed in detail in [2]. The equation is based on a generalization of the Maxwell model to the case of turbulent motion:

$$L_x^* \partial\overline{u'v'}/\partial x + L_y^* \partial\overline{u'v'}/\partial y + \overline{u'v'} = -\nu_t \partial\bar{u}/\partial y, \tag{1}$$

where L_x^* and L_y^* are the longitudinal and transverse relaxation paths, having the dimension of length; $\overline{u'v'}$ is the turbulent shear stress; $\bar{u}$ is velocity; ν_t is eddy viscosity; the superimposed bar denotes averaging over time.

In the solution of the relaxation equation, the quantities L_x^* and L_y^* are preassigned functions of the longitudinal coordinate x and transverse coordinate y even in the case of two-dimensional nongradient flow. However, this question has yet to be fully resolved, since the amount of reliable data now available on the laws governing the change in relaxation length is clearly inadequate. There is also no systematic data on the effect of the form of the source of perturbations and its relative dimensions on relaxation processes in shear flows. On the other hand, the study of the structure of a flow past different types of projections, irregularities, and obstacles is of interest in its own right – especially from the viewpoint of solving a whole range of practical problems [4, 5]. Thus, the tendency seen in recent years to account for relaxation phenomena in the analysis of nonequilibrium turbulent boundary layers requires more intensive study of the hydrodynamic structure of shear flows beyond perturbation sources of different geometries. Also required in this connection is determination of the laws which govern the change in the characteristic relaxation lengths under given conditions.

In the present study, we analyze the possibility of using the Hinze relaxation equation for a nonequilibrium turbulent shear flow which develops after the fluid crosses a circular

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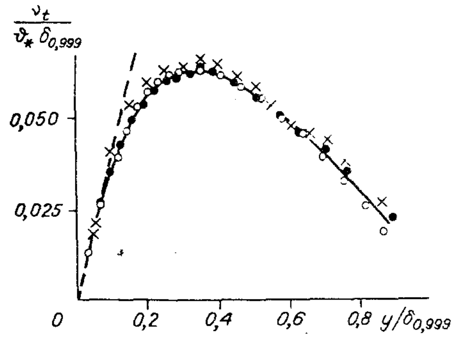

Fig. 1

cylinder in the transverse direction. In the case we will examine, the cylinder is located in the boundary layer of a flat plate.

The test were conducted on the T-324 low-turbulence subsonic wind tunnel at the ITPM SO AN SSSR (Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Academy of Sciencs of the USSR) [6]. The velocity of the undisturbed flow $U_\infty = 25$ m/sec, which corresponded to a Reynolds number $Re_1 = 1.66 \cdot 10^6$ m^{-1} at a distance of 1 m from the inlet. The measurements were made on a model in the form of a flat plate with the dimensions 2500 × 993 mm in plan. The thickness of the plate was 6 mm. The plate was positioned horizontally in the working part of the tunnel [7]. As the source of the disturbances, we used a circular cylinder positioned so that the flow moved tranversely relative to it. The cylinder was located in the developed turbulent boundary layer of the plate at a distance of about 600 mm from the leading edge. The relative diameter of the cylinder was discretely varied within the range $D/\delta_0 = 0.113$-0.388, while its position over the height of the layer was $\bar{y}_0 = y_0/\delta_0 = 0.094$-$0.94$, where δ_0 is the thickness of the boundary layer at the location of the cylinder. This thickness was equal to 10.6 mm.

We used an instrument complex made by the DISA company to measure mean velocity $\bar{u}$ at the test point of the flow field, the integral intensity of the velocity pulsations $\sqrt{\bar{u}'^2}$, and the turbulent shear stresses $\overline{u'v'}$ in the wake past the cylinder. Here, the transducers were miniature one- and two-wire anemometers with sensitive elements in the form of tungsten wires 5 μm in diameter and 1 mm in length and Wollaston wires having a working section 5 μm in diameter and 0.65 mm in length. The wires were prepared by the technology described in [8]. The experimental method and the results of certain procedural studes were detailed in [7]. We will stop here briefly to discuss the result from [7] that we deem to be the most important, since it summarizes the measurements of several parameters. We are speaking of the data on the distribution of dimensionless eddy viscosity ν_t over the height of the boundary layer. This data is shown in Fig. 1 for the longitudinal coordinates x = 1800 mm (dark circles) and 2108 mm (clear circles) in the case when the cylinder is absent as a source of perturbations. Here, $v_* = \sqrt{\tau_w/\rho}$ is dynamic velocity; $\delta_{0.999}$ is the thickness of the boundary layer calculated as the distance y from the plate surface, where $u/u_e = 0.999$ (the subscript e characterizes the external boundary of the layer: the superimposed bar denoting the averaging of velocity over time will henceforth be omitted for the sake of simplicity). The x's show experimental values of ν_t from [9], while the dashed line shows the change in the sought function in accordance with the linear Prandtl theory.

It is known that the quantity ν_t is extremely sensitive to the method of measurement and measurement errors and thus serves as reliable indicator of the correctness of the solution. The good agreement with [9] and the Prandtl theory in the boundary region of the flow is evidence of the reliability of the results and the possibility of using the given method to study the structure of the flow in the wake beyond the cylinder.

With the source of perturbations present, we obtained detailed measurements of the distribution of static pressure along the surface in the flow in the experiments. We also measured the magnitude and direction of the velocity vector of the shear flow and turbulence parameters at an average of 15 stations downstream from the surface. The resulting data allowed us to establish the pattern of development of the flow over the cylinder in the boundary layer of the plate. This pattern is schematized in Fig. 2 for $D/\delta_0 = 0.377$ and $\bar{y}_0 = 0.188$, i.e., when the cylinder was located directly on the surface.

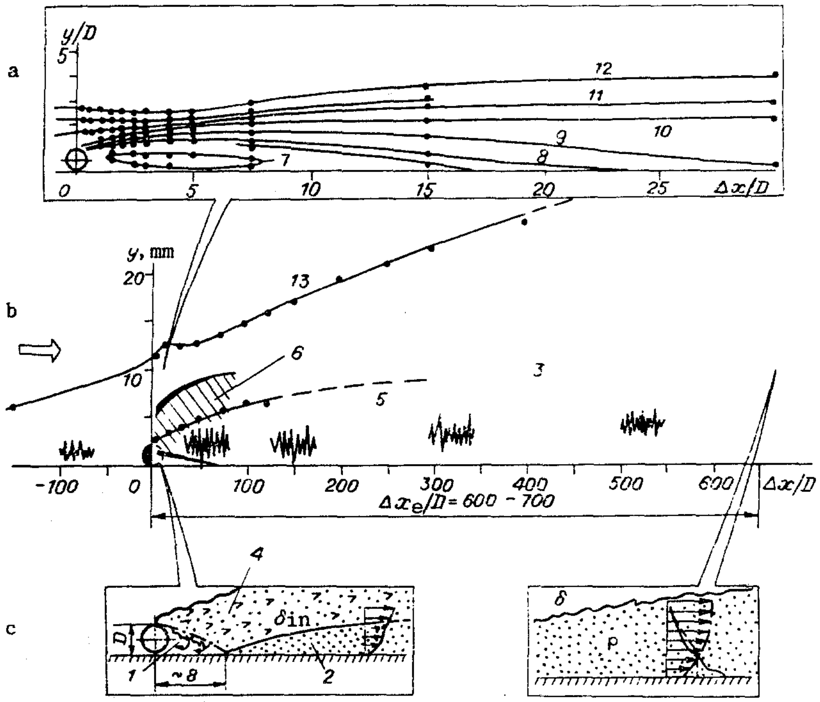

Fig. 2

On the whole, the hydrodynamic structure of the flow for this case is very similar to that seen in the case of flow past an obstacle of height H located on a flat surface [10]. Several characteristic stages of flow development can be discerned in this case: separation of the flow on the cylinder, the formation of a primary recirculation zone, and the formation of the main recirculation zone — the length of which $\Delta x/D$ corresponds to a value of approximately 8 as measured from the axis of the cylinder (region 1); attachment of separated flow to the surface of the plate and the formation of a new (internal) layer of thickness δ_{in} (region 2); relaxation of the flow to the state of complete equilibrium at $\Delta x_e/D \simeq (600\text{-}700)$ (region 3).

Complex mass transfer processes take place at each of these stages, these processes being connected both with each other and with external flow conditions (such as with the initial intensity of the perturbations dictated by the form and dimensions of the perturbation source). In particular, the interaction of the external flow and the flow in the recirculation region leads to the formation of a mixing layer (region 4) characterized by maximal momentum transfer $-\overline{u'v'}_{max}$. The position of the conditional axis of this layer along the investigated region is shown by line 5. The hatched region 6 shows the zone of maximum generation turbulence energy, characterized by the term $\left(-\overline{u'v'}\partial u/\partial y\right)D/u_e^3$. Here, $\partial u/\partial y$ is the gradient of mean velocity in the direction of the y axis. Within the indicated boundaries, the intensity of this quantity decreases by a factor of approximately 40 in the longitudinal direction. Meanwhile, the highest level of turbulence generation is observed immediately after the cylinder. This shows that the main source of turbulence is separation of the flow.

Let us discuss the presence of compression of the flow (throat) in the region $0 \leq \Delta x/D \leq 8$. This compression is evident when we look at the example of the distribution of lines of equal velocity u/u_e = const (see Fig. 2a, where lines 7 corresponds to u/u_e —0.1; 8-0.4; 9-0.6; 10-0.8; 11-0.9; 12-0.99). The effect under discussion actually means that there is an increase in flow velocity in the longitudinal direction, this increase being followed by a decrease. An analysis shows that the given phenomenon is due mainly to a rapid increase in the thickness of the boundary layer in the indicated region (line 13) and, thus, with a displacement that assists in the above-mentioned acceleration of the flow. Also quite visible here is the flow recirculation region, characterized by a negative sign of relative velocity u/u_e.

As regards the formation of a new (internal) layer, logarithmic and external regions develop along with the main regions in the velocity profiles. Evidence of this comes from

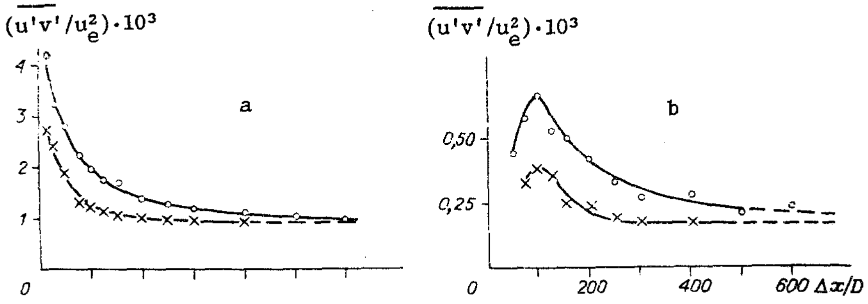

Fig. 3

the representation of velocity beyond the attachment point in the form of a wall law. Here, the external region of the new layer is the buffer region of the main layer. Signs of the presence of an internal layer can even be seen at a distance Δx corresponding to several tens of cylinder diameters.

In contrast to the near wake — in which the structure of the flow is determined to a considerable extent by speration phenomena — the character of flow below the point of attachment depends on the laws which govern the return of the parameters of the flow to the state of complete hydrodynamic equilibrium. These values of the parameters are denoted by the subscript e in Fig. 2. As is shown by data on the distribution of the disequilibrium parameter [7], this flow region is the longest region and has a length on the order of 600-700 cylinder diameters. A clear example of the flow relaxation of flow characteristics which takes place in this region is the distribution of dimensionless shear stress $\overline{u'v'}/u_e^2$ along the lines $y/\delta_{0.999} = 0.4$ (Fig. 3a) and 0.8 (Fig. 3b) (circles) beyond the cylinder $D/\delta_0 = 0.188$ at $\bar{y}_0 = 0.094$. Beginning with a certain distance $\Delta x/D$, the behavior of this relation becomes asymptotic. In accordance with heredity theory [2, 3], this region should be regarded as relaxational. Closer to the source of perturbations there is a certain transitional flow region characterized by a reduction in enthalpy $\overline{u'v'}/u_e^2$ with a decrease in $\Delta x/D$. The length of this region increases going toward the external boundary of the layer. In particular, at $y/\delta_{0.999} = 0.8$, the boundary of this region corresponds to the condition $\Delta x/D \approx 100$. Thus, from the viewpoint of such characteristics as turbulent shear stress and with respect to the construction of an appropriate theoretical model, the entire nonequilibrium flow region beyond the cylinder can be conditionally divided into three characteristic regions (the new layer not included): near wake; transitional region; relaxation region.

Figure 3 also shows results of calculations (circles) performed within the framework of the Boussinesq model: $\overline{u'v} = \nu_t \partial u/\partial y$. Eddy viscosity ν_t was calculated with the use of the two-layer Prandtl—Clauser model [11]: $\nu_t = \varkappa^2 y^2 D(y)\partial u/\partial y$ in the internal region of the boundary layer; $\nu_t = \gamma K u_e \delta^*$ in the external region. Here, D(y) is the van Dreist damping factor; $\varkappa = 0.4$ is the Kármán constant; K = 0.0168 is an empirical coefficient; δ^* is the displacement thickness; u_e is velocity on the external boundary of the boundary layer; $\gamma = [1 + C(y/\delta)b]^{-1}$(C = 6 and b = 7.5 are empirical coefficients obtained from an analysis of the experimental data in [9]). The derivative $\partial u/\partial y$ was determined by graphic differentiation of experimental profiles of velocity at the analyzed point of the flow field. The boundary between the internal and external regions of the boundary layer was found on the basis of the best agreement between the results obtained from the Prandtl theory and the Clauser hypothesis for the same value of the transverse coordinate y.

It is evident that, in contrast to the case of an equilibrium flow (where the local two-layer model satisfactorily describes the distribution of turbulent stresses in the boundary layer [12]), similar results in the relaxation region show that the given model is almost completely unsuited for predicting nonequilibrium flows. Under the conditions we have studied here, the difference between the experimental and theoretical values of $\overline{u'v'}/u_e^2$ is 30-50%, depending on the relative coordinate $y/\delta_{0.999}$. This difference naturally becomes smaller as we increase the relative distance $\Delta x/D$. We should point out that the deviation of the theoretical results from the experimental data increases with an increase in the dimensionless coordinate $y/\delta_{0.999}$ (see Fig. 3a and b). This means that the relative contribution of memory effects increases in the direction of the external boundary

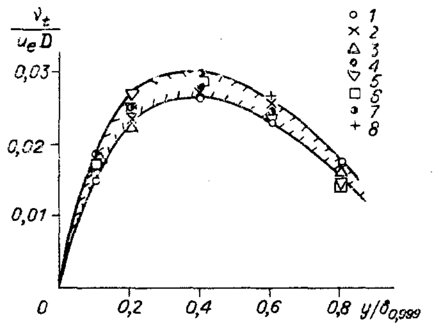

Fig. 4

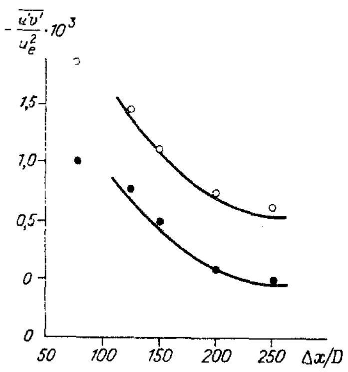

Fig. 5

of the boundary layer. In other words, large-scale turbulence has a greater capacity to remember previous flow states, i.e., it has a "longer" memory for perturbations generated by a two-dimensional source. Thus, when calculations such as the above are being performed, it is necessary to account for hereditary characteristics of the flow or its history.

As was noted above, one way of simplifying the evaluation of flow memory is to use the relaxation equation (1). Assuming that $L_x^* \gg L_y^*$, we solved the equation for eddy viscosity ν_t. The latter quantity was determined in such a way that, by appropriately selecting the relation for the longitudinal relaxation length L_x^*, we ensured that the relationship between the turbulence characteristics and the mean velocity over the height of the boundary layer was universal. An example of this approach is shown in Fig. 4 for the case of flow past a cylinder $D/\delta_0 = 0.188$ placed directly on the surface of a plate. Here, $\bar{y}_0 = 0.094$. The points 1-8 in the figure correspond to $\Delta x/D$ = 30, 50, 75, 100, 155, 200, 300, 400. It can be seen that within a certain spread (shown by the hatched boundaries) there is a single relation $\nu_t/u_e D = f(y/\delta_{0,999})$, for different $\Delta x/D$. In this range, the longitudinal relaxation length is expressed in the form $L_x^* = a\Delta x$ $(a \approx 0{,}4)$. The latter us evidence of the presence of similarity in the relations between the turbulence charcteristics and the mean velocity profile in the nonequilibrium flow region. Only in the central part of the layer is the universal character of the relations disturbed somewhat. This occurs because the quantity L_x^* was assumed to be constant over the height of the layer. Strictly speaking, it is not, since the coefficient a actually increases slightly in the direction of the external boundary of the layer and the function $L_x^* = f(\Delta x)$ is nonlinear. However, use of the given relation in linear form is acceptable for approximate practical calculations.

A more important question in our minds is whether the coefficient a changes or remains constant for two-dimensional sources of different geometries and relative dimensions. In connection with this, we used the value of L_x^* obtained with a = 0.4 to calculate the turbulent shear stresses $\overline{u'v'}$ directly from the Hinze relaxation equation in the wake after a large-diameter cylinder ($D/\delta_0 = 0.377$) at $\bar{y}_0 = 0.188$. The results of these calculations are shown in Fig. 5 (line) in the form of the relation $\overline{u'v'}/u_e^2 = f(\Delta x/d)$ for two values of the transverse coordinate $y/\delta_{0.99}$, where the role of flow history is important. The light ($y/\delta_{0.99} = 0.7$) and dark ($y/\delta_{0.99} = 0.8$) circles show data from measurements of the shear stresses. The lack of theoretical data for $\Delta x/D \lesssim 100$ is due to the existence of a transitional flow region in which the hereditary model [2, 3] is invalid. On the other hand, in the relaxation region ($\Delta x/D \gtrsim 100$) the theoretical values of $\overline{u'v'}/u_e^2$ agree satisfactorily with the measurement results with an error no greater than about 10%. Such a deviation can probably be considered acceptable for such turbulence characteristics. It is also very significant that an expression for L_x^* with a coefficient a close to 0.4 was obtained experimentally in [10] for flow past a rectangular obstacle with a relative height $\bar{H} = H/\delta_0 \lesssim 0.25$ when it was placed on a flat surface. All this suggests that the relation for longitudinal relaxation length does not change in the case of flow past sources of perturbations with a relative height $D/\delta_0 \lesssim 0.4$ and depends only slightly or not at all on the shape of the source — at least at the flow velocities studied here. Thus, the expression for L_x^* presented above is valid in practical calculations of turbulent shear stresses in nonequilibrium flows based on the Hinze relaxation equation. However, in order to reach a definitive conclusion regarding this matter, it will be necessary to obtains systematic data for sources of perturbations of different shapes.

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